A_1 The coordinates of the midpoints of two adjacent sides of a square are $P(-2, 5)$ and $Q(1, 2)$. Find the area of the square.

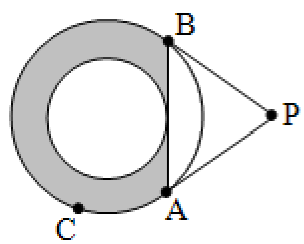
A_2 The difference between $\frac{A}{7}$ and $\frac{B}{9}$, where *A* and *B* are positive integers, is $\frac{43}{63}$. Find the ordered pair (*A*, *B*) so that *A* + *B* is as small as possible.

B_1 In terms of *n*, find the fourth of seven consecutive positive odd integers whose sum is 7^{n+1}.

B_2 If *a*, *b*, *c*, and *d* are positive integers such that $a^5 = b^6, c^3 = d^4$ and $d - a = 61$. Find the smallest value of $c - b$.

C_1 A certain fraction can be expressed as either $.\overline{3}$ in base *x* or $.\overline{4}$ in base *y*, where *x* and *y* are positive integers. Find the numeric value of $6y-8x$.

C_2 The circles are concentric and $m\widehat{ACB}=240^\circ$. Chord $\overline{AB}$ is tangent to the smaller circle and $\overline{PB}$ and $\overline{PA}$ are tangent to the larger circle.
If $PB=8\sqrt{3}$, find the area of the shaded region.



D_1 When 3200 people attended a concert, the stadium was $(x+2)\%$ full. When 6100 people attended the same concert the next night the stadium was $(3x-1)\%$ full. How many people would be in attendance if the stadium was 100% full?

D_2 Find all <u>exact</u> values for $\frac{x}{2y}$ which satisfy the equation

$$\frac{x}{2y}+\sqrt{\frac{2y}{x}}=2$$

E_1 Given that $\log_a b^2 - \log_b a^2 = 1$, compute all numerical values of $\log_a b$.

E_2 When a two-digit positive base 10 integer N is multiplied by the number which is two more than the units' digit, the product has three digits each of which is the tens digit of the original number. What is N?

F_1 A positive integer N is written in bases 4, 5 and 9. In at least one of bases 5 and 9, the representation of N has 3 significant digits, but in base 4, the representation of N has 4 significant digits. How many integers N satisfy these conditions?
[For the purposes of this problem, the integers 001, 30 and 201, regardless of base, have 1, 2 and 3 significant digits respectively.]

F_2 Find all ordered triples of integers (x, y, z) which satisfy the following system: $yz - 3(y+z) = 6$; $xy - 3(x+y) = 11$; $xz - 3(x+z) = 3$

G_1 Let A_1 and A_2 be distinct two-digit prime numbers with the following properties:

i. The sum of the digits of A_1 is a prime number B_1
ii. The sum of the digits of A_2 is a prime number B_2
iii. $B_1 + B_2$ is equal to the prime number C.
iv. The sum of the digits of C is a composite number.

Find all possible values of the sum $A_1 + A_2$.

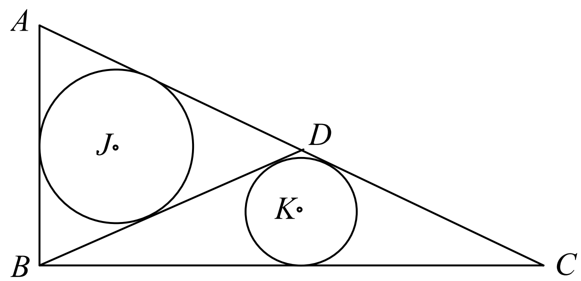
G_2 A, B and C are acute angles such that $\sin A = \dfrac{1}{2}$, $\cos B = \sqrt{1-k^2}$ and $\tan C = \dfrac{12}{5}$. If, additionally, $\cos A \cdot \sin B \cdot \csc C = \dfrac{1}{2}$, determine the exact value of $\cos B$.

H_1 Given: $3^6 \cdot 3^8 \cdot 3^{10} \cdot 3^{12} \cdots 3^{2n} \cdot 3^{2n+2} = 3^{594}$
Find the value of n.

H_2 A, B and C are three non-similar regular polygons with congruent sides. Polygon A is a heptagon, i.e. it has 7 sides. A, B and C can tessellate the plane, i.e. they can be joined at a common vertex and completely surround that point without overlapping. What is the sum of the number of sides in polygons B and C?

I_1 The first term of a geometric sequence is $(-a+bi)$, where a and b are positive integers. If the second and third terms are the roots of $x^2-2x+2=0$, then compute the sum of the first ten terms of the sequence.

I_2 In ΔABC, $AB = 10$, $BC = 24$, and $AC = 26$. $\overline{BD}$ is a median and circles J and K are inscribed in ΔABD and ΔDBC respectively. Determine the ratio of the area of circle J to the area of circle K.



J_1 If 2 is added to each of the roots of the equation $x^3+Ax^2+Bx-12=0$, the resulting equation is $x^3-3x^2-3x+k=0$. Find the ordered triple of integers (A, B, k) that satisfies all of the above conditions.

J_2 Given: $\{x \mid 2^{4x}+48=2^{2x+4}, x\in\mathbb{R}\}$. Determine all real values of x. If necessary, express answers in the form $\log_b N$, where N is a real number.

Answer Key - ARML Tryout Questions – 2007

A1	36	G1	40, 58, 94, 100
A2	$(3, 10)$	G2	$\frac{11}{13}$
B1	7^n	H1	23
B2	593	H2	45
C1	-2	I1	$2i$
C2	$\frac{80\pi}{3}+16\sqrt{3}$	I2	625 : 324
D1	50,000	J1	$(3, -3, -2)$
D2	1 or $\frac{3-\sqrt{5}}{2}$	J2	1, $\log_2\sqrt{12}$ ($\log_4 12$ or $\log_2(2\sqrt{3})$)
E1	$\frac{1\pm\sqrt{17}}{4}$		
E2	37		
F1	192		
F2	$(7,8,6)$ or $(-1,-2,0)$		